6.37 a) $z^2 = 5 + 12i$

a) Ans $z = a + bi \Rightarrow (a+bi)^2 = 5 + 12i \Leftrightarrow a^2 + 2abi - b^2 = 5 + 12i$.

Jmf real- & imaginärdel

$$\begin{cases} a^2 - b^2 = 5 & (i) \\ 2ab = 12 & (ii) \\ a^2 + b^2 = 13 & (iii) \end{cases}$$

$|z^2| = |5+12i| = 13 \Rightarrow$

i) + iii) ger $2a^2 = 18 \Leftrightarrow a = \pm 3$

ii) ger $2 \cdot 3 \cdot b = 12 \Rightarrow b = \pm 2$

Ekv (2) ger $\pm(a+bi)$ samma tecken.

$z = \pm(3 + 2i)$

637b) $z^2-(2+2i)z-5-10i=0$ Kvadratkomplettera!

$$z^2-(2+2i)z-5-10i=0 \Leftrightarrow \underbrace{\left(z-\frac{2+2i}{2}\right)}_{w}^2-\left(\frac{2+2i}{2}\right)^2-5-10i=0 \Leftrightarrow$$

$$w^2-\frac{4+8i-4}{4}-5-10i=0 \Leftrightarrow w^2-2i-5-10i=0 \Leftrightarrow w^2=12i+5$$

Sätt $w=a+bi$ & få $w^2=12i+5 \Leftrightarrow (a+bi)^2=12i+5 \Leftrightarrow a^2+2abi-b^2=12i+5 \Leftrightarrow$

$$\begin{aligned} a^2-b^2 &= 5 \quad (1) \\ 2ab &= 12 \quad (2) \\ a^2+b^2 &= 13 \quad (3) \end{aligned}$$

$(1)+(3) \Rightarrow 2a^2=18 \Leftrightarrow a=\pm 3$

$(3)-(1) \Rightarrow 2b^2=8 \Leftrightarrow b=\pm 2$

Tecken? $(2) \Rightarrow$ Samma!

$w=\pm(3+2i) \Rightarrow w_1=3+2i$ & $w_2=-3-2i$ Åter till z

$w=z-\frac{2+2i}{2}$ Ins. $w_1 \Rightarrow 3+2i=z_1-\frac{2+2i}{2} \Leftrightarrow z_1=3+2i+\frac{2+2i}{2}=4+3i$

$w_2 \Rightarrow -3-2i=z_2-\frac{2+2i}{2} \Leftrightarrow z_2=-3-2i+\frac{2+2i}{2}=-2-i$

540 $(2+i)z^2+(1-7i)z-5=0 \iff (2+i)z^2+(1-7i)z=5 \iff$

$$z^2+\frac{(1-7i)z}{2+i}=\frac{5}{2+i} \iff z^2+\frac{z(1-7i)(2-i)}{(2+i)(2-i)}=\frac{5(2-i)}{(2+i)(2-i)} \iff$$

$$z^2+z\left(\frac{2-i-14i-7}{5}\right)=2-i \iff z^2+z(-1-3i)=2-i \iff$$

$$z^2-(1+3i)z=2-i \iff \Big(\underbrace{z-\frac{1+3i}{2}}_{w}\Big)^2=2-i+\left(\frac{1+3i}{2}\right)^2 \iff$$

$$w^2=\frac{1}{4}(8-4i)+\frac{1}{4}(1+6i-9)=\frac{1}{4}(2i)=\frac{i}{2} \iff w^2=\frac{i}{2} \iff \quad w=a+bi$$

$$(a+bi)^2=\frac{1}{2}i \iff a^2-b^2+2abi=\frac{1}{2}i \Rightarrow \begin{cases} a^2-b^2=0 & (1) \\ 2ab=\frac{1}{2} & (2) \\ a^2+b^2=\frac{1}{2} & (3) \end{cases}$$

$$|w^2|=\frac{1}{2} \Rightarrow a^2+b^2=\frac{1}{2}$$

(1)+(3) $2a^2=\frac{1}{2} \iff a=\pm\frac{1}{2}$; (3)−(1) $2b^2=\frac{1}{2} \iff b=\pm\frac{1}{2}$

(2) $\Rightarrow$ $a>0$ & $b>0$ eller $a<0$ & $b<0$. $\Rightarrow w=\pm\left(\frac{1}{2}+\frac{1}{2}i\right)$

$$\Big(\underbrace{z - \frac{1+3i}{2}}_{w}\Big)^2 =$$

$$z = a+bi$$

$$w_1: \quad z - \frac{1+3i}{2} = \frac{1}{2} + \frac{1}{2}i \iff a+bi = \frac{1}{2} + \frac{1}{2}i + \frac{1}{2} + \frac{3}{2}i = 1 + 2i$$

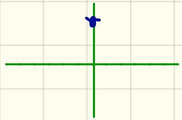
$$z = a+bi$$

$$w_2: \quad z - \frac{1+3i}{2} = -\frac{1}{2} - \frac{1}{2}i \iff a+bi = -\frac{1}{2} - \frac{1}{2}i + \frac{1}{2} + \frac{3}{2}i = i$$

6.41 a) & b) a) $z^3 = i$ $z = a + bi = re^{i\theta}$ Binomisk ekv: $z^n = w$

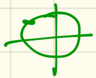
$z^3 = i \Leftrightarrow (re^{i\theta})^3 = i \Leftrightarrow re^{i3\theta} = i$ $|VL| = |HL| \Rightarrow r = 1$

$e^{i3\theta} = i$

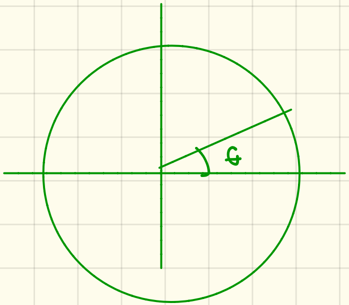


$e^{i\pi/6} = \underbrace{\cos \pi/6}_{\sqrt{3}/2} + i \underbrace{\sin \pi/6}_{1/2} = \sqrt{3}/2 + \frac{1}{2}i$

$3\theta = \frac{\pi}{2} + 2k\pi \Leftrightarrow \theta = \frac{\pi}{6} + k\frac{2}{3}\pi$

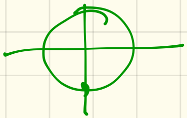


$k = 0 \Rightarrow \sqrt{3}/2 + 1/2\, i$



$k = 1 \Rightarrow \theta = \pi/6 + \frac{2\pi}{3} = \frac{5\pi}{6} \Rightarrow -\sqrt{3}/2 + \frac{1}{2}i$

$k = 2 \Rightarrow \theta = \frac{\pi}{6} + \frac{4}{3}\pi = \frac{9}{6}\pi = \frac{3}{2}\pi \Rightarrow e^{\frac{3}{2}\pi i} = -i$



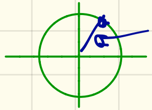
6.4 (b)

$z^3 = 1+i$

De Moivres formel: $(e^{i\theta})^n = e^{i\theta n}$

Ansats: $z = re^{i\theta} \Rightarrow z^3 = 1+i \Leftrightarrow r^3 e^{i3\theta} = 1+i$

$r^3 = |1+i| = |\sqrt{2}| \Rightarrow r = 2^{1/2 \cdot 1/3} = 2^{1/6}$



$\theta = \pi/4$

$3\theta = \pi/4 + 2k\pi, \ k \in \mathbb{Z}$

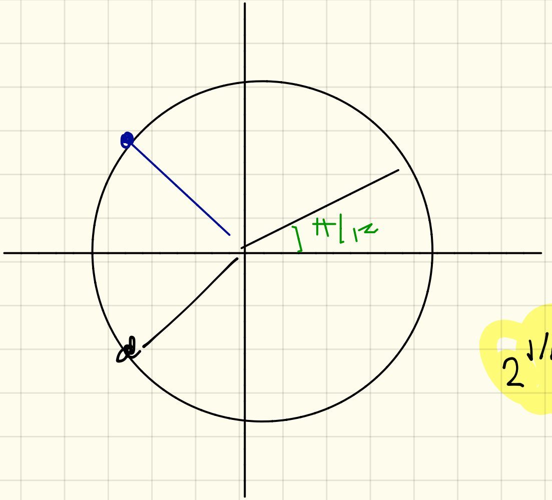
$\theta = \pi/12 + \frac{2}{3} k\pi$

$k=0$: $\pi/12$

$k=1$ $\pi/12 + \frac{2}{3}\pi = \frac{9\pi}{12} = \frac{3}{4}\pi$

$k=2$ $\pi/12 + \frac{4}{3}\pi = \frac{17\pi}{12} = \pi + \frac{5\pi}{12}$

$k=3$ $\pi/12 + \frac{6}{3}\pi = \pi/12 + 2\pi.$



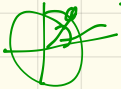
$2^{1/6} e^{i\,3/4\pi}, \ 2^{1/6} e^{i\frac{17\pi}{12}}, \ e^{i\,\pi/12}$

6.42

$z^6 - 2z^3 + 2 = 0 \underset{z^3 = w}{\Leftrightarrow} w^2 - 2w + 2 = 0 \Leftrightarrow w = 1 \pm \sqrt{1-2} \Leftrightarrow$

$w_1 = 1 + i$
$w_2 = 1 - i$ $\Rightarrow z^3 = 1+i$ eller $1-i$

$z^3 = 1+i$: $r^3 e^{i3\theta} = 1+i$, $|1+i| = \sqrt{2}$

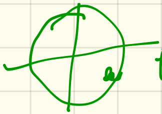


$\pi/4$

$r^3 = \sqrt{2} \Leftrightarrow$

$2^{1/6} e^{i\,3/4\pi}, 2^{1/6} e^{i\frac{17\pi}{12}}, e^{i\pi/12}$

$z^3 = 1-i$ $r^3 e^{i3\theta} = 1-i$, $r = 2^{1/6}$



$\theta = -\pi/4 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$3\theta = \frac{7\pi}{4} + 2k\pi$, $k = 0, 1, 2.$

$\theta = \frac{7\pi}{12} + \frac{2}{3}k\pi$ $\underline{k=0}$ $\frac{7\pi}{12}$; $\underline{k=1}$ $\frac{7\pi}{12} + \frac{2}{3}\pi = \frac{15\pi}{12} = \frac{5}{4}\pi$

$\underline{k=2}$ $\frac{7\pi}{12} + \frac{4}{3}\pi = \frac{23}{12}\pi$

6.44 $\varphi(x) = x^3 - 2x^2 - 19x + \alpha$, best α. het fakt $(x-1)$

$$
\begin{array}{l|l}
x^3 - 2x^2 - 19x + \alpha & x-1 \\
-x^2(x-1) & x^2 - x - 20 \\
\hline
-x^2 - 19x + \alpha & \\
+x(x-1) & \\
\hline
-20x + \alpha & \\
+20(x-1) & \\
\hline
\alpha - 20 &
\end{array}
$$

$\alpha - 20 = 0 \Leftrightarrow \alpha = 20$

$x^2 - x - 20 = 0 \Leftrightarrow \frac{1}{2} \pm \sqrt{\underbrace{\left(\frac{1}{2}\right)^2 + 20}_{\frac{1}{4}+20 = \frac{81}{4}}} = \frac{1}{2} \pm \frac{9}{2}$ $\quad x_1 = \frac{1}{2} + \frac{9}{2} = \frac{10}{2} = 5$

$x_2 = \frac{1}{2} - \frac{9}{2} = \frac{-8}{2} = -4$

$\varphi(x) = (x-1)(x-5)(x+4)$

6.45 $p(z) = z^4 - 2z^3 + 2z^2 - 10z + 25 = 0$, rötter $z = 2+i$

$z = -1-2i$

Enl. sats ä rötterna även $2-i$ & $-1+2i$

6.46. Ge gr. 6 pol. enkelt nollst $z = 2 - i$ & dubbelt i $z = i$

$$(z-(2-i))(z-(2+i))(z-i)^2(z+i)^2$$

Dubbelt nollställe $\Rightarrow$ kvadrera.

6.49 $z^4 - z^3 + 7z^2 - 9z - 18 = 0$, har rot $z = bi$, $b \in \mathbb{R}$

$(bi)^4 - (bi)^3 + 7(bi)^2 - 9(bi) - 18 = 0 \Leftrightarrow$

$b^4 + b^3 i - 7b^2 - 9bi = 18 \Leftrightarrow (b^4 - 7b^2) + (b^3 - 9b)i = 18$

① $b^4 - 7b^2 = 18$

② $b^3 - 9b = 0 \Leftrightarrow b(b^2 - 9) = 0 \Leftrightarrow b = 0, 3, -3$

$b = 0$: ① $\Rightarrow 0 - 0 \neq 18$, $b = 3$: ① $3^4 - 7 \cdot 3^2 \overset{?}{=} 18 \Leftrightarrow 81 - 63 = 18$ ok

$b = -3$: ① $\Rightarrow -3^4 - 7 \cdot -3^2 = 81 - 63 = 18$ ± 3 ok

$z_1 = 3i$, $z_2 = -3i$ resten? Gör div!

$(z - 3i)(z + 3i) = z^2 + 3zi - 3iz + 9 = z^2 + 9$

6.49 forts...

$$z^2 - z - 2$$

$$z^4 - z^3 + 7z^2 - 9z - 18 \;\Big|\; z^2 + 9$$

$$-z^4 - 9z^2$$

$$-z^3 - 2z^2 - 9z - 18$$

$$+z^3 + 9z$$

$$-2z^2 - 18$$

$$+2z^2 + 18$$

$$0 \quad :-)$$

$$z^2 - z - 2 = 0 \Leftrightarrow z = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 2} = \frac{1}{2} \pm \sqrt{\frac{9}{4}} = \frac{1}{2} \pm \frac{3}{2}$$

$$z_3 = \frac{1}{2} + \frac{3}{2} = 2$$

$$z_4 = \frac{1}{2} - \frac{3}{2} = -\frac{2}{2} = -1$$

$$z_1 = 3i,$$

$$z_2 = -3i;$$